Key

Math 4

Name_____________________________

5-3 The Derivative at a Point

Date_________

In this Activity, you will be working towards the following learning goals:
I can use the definition of derivative to compute derivatives
I can use derivatives and their graphs to identify properties of functions

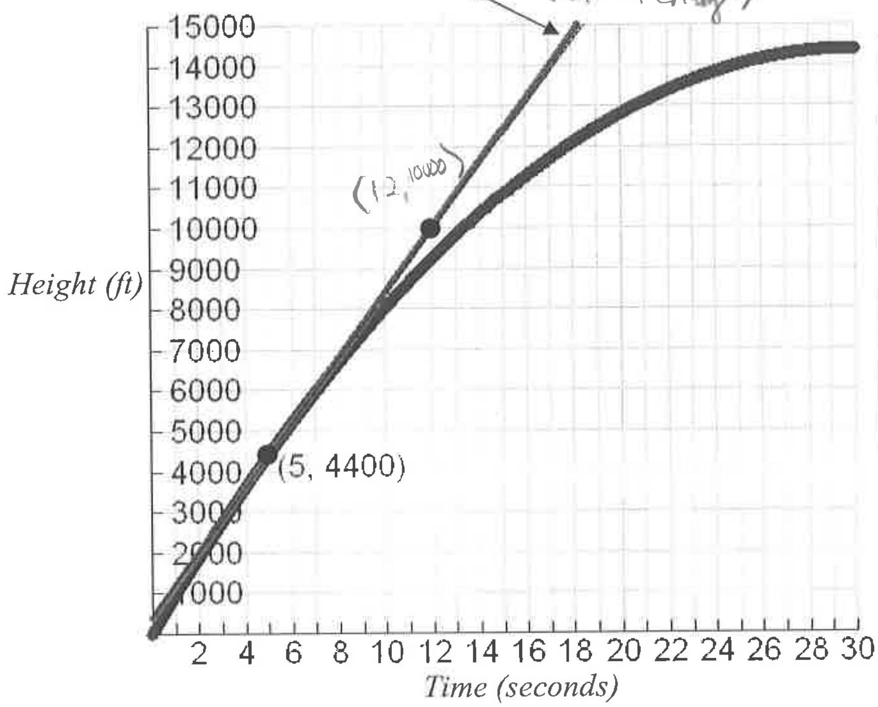
I. The projectile referred to in Investigation 1 had a position equation of $h(t) = 960t - 16t^2$. Part of the path of the projectile is shown below. In addition, secant lines representing your computations from parts a – d have been drawn. Following the example ($\Delta t = 1$, $\Delta t = 0.5$, $\Delta t = 0.1$, $\Delta t = 0.01$)

A. What is happening to the distances between the points on the curve that are connected by the secant lines as Δt gets smaller?

GSP file

Eventually the 2 points become one; what results is a **tangent line**. *The slope of the tangent line is the instantaneous velocity of the object at that point in time.*

tangent line (We will talk more about tangent lines as a class.) Rate of Change



Calculate the slope of the tangent line when $t = 5$.

$$m = \frac{10000 - 4400}{12 - 5} = \frac{5600}{7} = \boxed{800 \text{ ft/s}}$$

Look back at your limit statement from investigation 1. Do the numbers match? Yes!

B. The example in part A illustrates the geometric definition of **instantaneous velocity**. Here is the algebraic definition:

Suppose an object is moving so that each time t is at a position $f(t)$ along a line. The **instantaneous velocity** is the limit as $\Delta t \to 0$ of the average velocity of the object between times t and $t + \Delta t$. In symbols, this means:

$$\textbf{Instantaneous Velocity} = \lim_{\Delta t \to 0} \frac{f(t+\Delta t) - f(t)}{\Delta t}$$

Recall: The formula for the average velocity of the projectile from Investigation 1:

$$AV = 960 - 32t - 16\Delta t$$

Use the formula to find the *instantaneous velocity* of the projectile at $t = 5$ seconds.

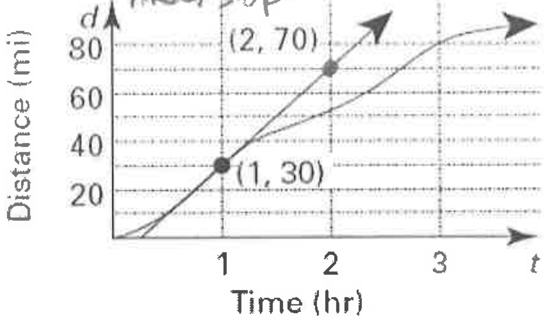
Finish: $IV = \lim_{\Delta t \to 0} [960 - 32t - 16\Delta t]$

$$= \lim_{\Delta t \to 0} (960 - 32(5) - 16\Delta t)$$

$$= \lim_{\Delta t \to 0} (800 - 16\Delta t) \to = 800 - 16(0) = \boxed{800 \text{ ft/s}}$$

C. The graph below shows the distance d in miles traveled by a car in t hours after it begins a trip. Find the **instantaneous velocity** of the car at $t = 1$ hour.

AROC/Slope



$$v = \frac{70-30}{2-1} = \boxed{40 \text{ mph}}$$

II. The previous examples used the quantity $\lim\limits_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$. This quantity arises in many other settings & so it has been given a special name. It is called the **derivative of f at x.**

Definition: The **derivative of a real function f at x,** denoted **$f'(x)$** is:

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Note: The function must be continuous & smooth at point x.

Example: Let $f(x) = x^2 - 3$

Method A: Algebraic

Follow these steps:
1. Use the definition to derive a formula for f'
2. Evaluate your formula when $x = 2$.

1.) $$f'(x) = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - 3 - (x^2 - 3)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{x^2} + 2x\Delta x + \Delta x^2 \cancel{- 3} \cancel{- x^2} \cancel{+ 3}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x\Delta x + \Delta x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} 2x + \Delta x$$

$$= 2x + 0 = \boxed{2x}$$

2.) $f'(2) = 2(2)$

$= \boxed{4}$

Method B: Graphical TI-nspire™*-style*

Follow these steps:

1. In a new document (NOT "Scratchpad") graph the function: $f(x) = x^2 - 3$.
2. If necessary, adjust the window to see the parabola.
3. Press **[MENU]→8: Geometry→1 :Points & Lines→7: Tangent** to open the Tangent Line tool.
4. Click **[CLICK]** on the function graph, then press [CLICK] again to construct the tangent line.
5. Press **[ESC]** to exit the Tangent Line tool.
6. Press Press **[MENU]→1: Actions→8 :Coordinates and Equations**. **[CLICK]** on the point of tangency. The coordinate of the point should now be displayed.
7. Hold the **[CLICK]** until the hand closes. Drag the point of tangency. Note what happens to the slope of the tangent line as the point of tangency moves along the function.
8. Press **[ESC]** to "let go" of the point of tangency.
9. Double **[CLICK]** on the *x*-coordinate of the displayed point; delete the *x*-coordinate and enter and *x*-coordinate of 2; **[CLICK** to enter the coordinate.
7. ~~Select the tangent line and then~~ press **[MENU] → 8: Geometry→3: Measurement→3: Slope**. **[CLICK** on the tangent line. The slope should be displayed. → or just look @ the equation of the tangent line.
8. Press **[ESC]** to exit the Slope tool.

What is the slope of the tangent line at x = 2?

M = 4 → same as the previous page!

Your answer should be the same using both methods!